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OPTIMAL FREQUENCY SEPARATION IN CYLINDRICAL SHELLS.(U)

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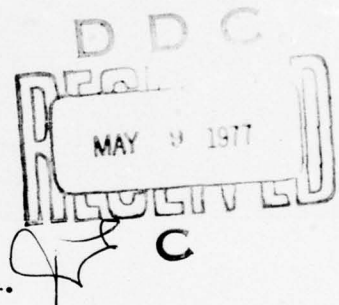
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OPTIMAL FREQUENCY SEPARATION IN  
CYLINDRICAL SHELLS

by

Michael Pappas



March 1977

NJIT Report No. NV 11

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# ABSTRACT

The problem of optimal frequency separation involves the simultaneous separation of several vibration modes even where only maximal frequency separation between the two lowest frequencies is desired. Conventional optimization procedures which maximize a single function cannot deal with such a problem. Earlier attempts to treat this problem have therefore encountered difficulty in solution. New optimization methodology is, therefore, introduced which allows the simultaneous separation of several modes. This new procedure is coupled with a powerful new general optimization algorithm. Test results indicate that this new combined procedure is capable of the reliable solution of the optimal frequency separation problem.

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## I. Introduction

Bronowicki et al., [1,2] in their extension of earlier studies of optimal synthesis of "T" ring stiffened cylindrical shells under hydrostatic pressure [3,5] introduce the problem of optimal natural frequency separation. Their pioneering effort uncovered several difficulties associated with such problems.

Ref. [1] in its discussion of their type III problem (separation of the two lowest frequencies with primarily axial motion) notes that their optimal search terminates at a design where the second primarily "axial" frequency has a radial component that is equal to the axial component. In the type II problem (separation of the two lowest frequencies) their search terminates at designs where the second and third frequencies are essentially equal. The authors of [1] recognize the difficulty in obtaining an optimal solution to the axial frequency separation problem which they attribute to the need for an improved definition of primarily axial frequencies.

The difficulty with these problem types is that a conventional mathematical programming formulation and solution procedure is not effective at the points described above. In problem type II at points where the second and third lowest frequency are essentially equal it is necessary to separate the first and third as well as the first and second frequencies in order to continue the search. In problem type III it is necessary to suppress growth of the radial component associated with the second primarily axial frequency in addition to increasing its separation from the lowest axial frequency in order to continue movement toward the optimum.

In addition to the above difficulties Ref. [1] utilizes an optimization procedure which is apparently unreliable on the problems formulated therein.



This report presents new methodology for treating such problems. It applies this method to problem type II of Ref. [1] as well as a similar problem so as to test its effectiveness and to gain additional insight into optimal frequency separation. This work utilizes a new and powerful mathematical programming procedure [6], which appears to overcome the reliability problem associated with that used in Ref. [1] thereby allowing generation of more accurate data on the characteristics of designs with frequency separation.

In addition, this improved optimization procedure is adapted to a minimum weight problem formulation similar to that of Ref. [3] and problem type I of Ref. [1] (minimum weight with minimum natural frequency constraint) to demonstrate its effectiveness on a six variable form of this problem. Previous attempts to solve this six variable form had proven to be unsuccessful [3,7].

## II. Problem Formulation

Mathematical programming procedures treat the problem:

Find those values  $\bar{x}_i$  of the variables  $x_i$  that result in

$$f(\bar{x}_i) = \min f(x_i) \quad i = 1, 2, \dots, I \quad (1)$$

subject to the constraints

$$g_j(x_i) \geq 0 \quad j = 1, 2, \dots, J \quad (2)$$

and

$$x_i^{\ell} \leq x_i \leq x_i^u \quad (3)$$

These procedures start from some initial point  $x_i^0$  and by some strategy, usually based on the local properties the functions involved, generate a sequence of points where

$$f(x_i^{r+1}) < f(x_i^r) . \quad (4)$$

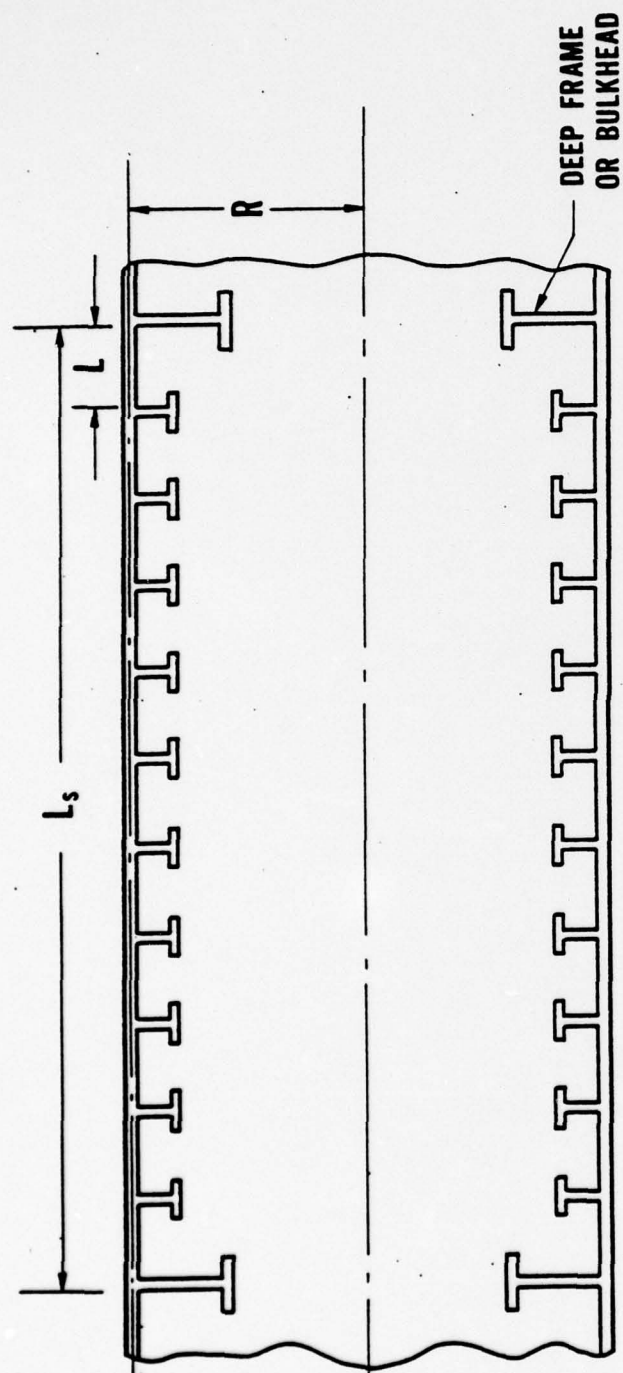
In problem type II of Ref. [1]

$$f(x_i) = \omega_1 - \omega_2 \quad (5)$$

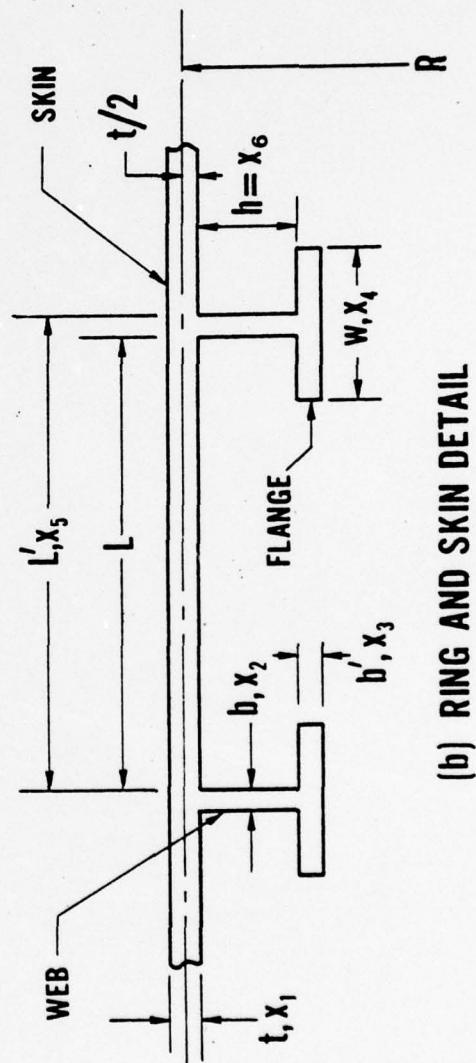
where  $\omega_1$  and  $\omega_2$  are the first and second in vacuo, natural frequencies of the shell segment shown in Figure 1. The variables  $x_i$  are the skin, stiffener web and stiffener flange thicknesses, the flange width, stiffener spacing and stiffener web height respectively as given in Figure 1.

Consider the problem of locating an improved point in the neighborhood of a point  $x_i^r$  where

$$\omega_2 \approx \omega_3$$



**(a) SHELL SEGMENT CROSS-SECTION**



### (b) RING AND SKIN DETAIL

Figure 1. Shell Cross-Section

and where  $\omega_3$  is the third natural frequency. Each frequency  $\omega_k$  where  $k = 1, 2, 3 - \infty$  and

$$\omega_1 \leq \omega_2 \leq \omega_3 \leq \dots \quad (6)$$

is a function of the shell design parameters such as its radius, length, material properties, applied hydrostatic pressure, etc., the problem variables  $x_1^r$  (refer to Fig. 1) and the number of circumferential waves  $n_k^r$  and axial half-waves  $m_k^r$  associated with frequency  $\omega_k^r$  at point  $x_1^r$ . Thus, for a given set of design parameters the  $k^{\text{th}}$  natural frequency at point  $x_1^r$  is

$$\omega_k^r = \omega(x_1^r, n_k^r, m_k^r) \quad (7)$$

a function of both continuous and integer variables. Gradient based procedures such as those used in [1] determine a direction for function improvement from the derivatives of  $\omega_1^r$  and  $\omega_2^r$  with respect to  $x_1^r$ . An attempt to move in a direction increasing the separation between the modes associated with these frequencies where  $\omega_2 \approx \omega_3$  will often reduce the separation between the modes associated with  $\omega_1^r$  and  $\omega_3^r$ . These modes will then produce the two lowest frequencies after a move to point  $x_1^{r+1}$  where now the separation between these frequencies will be lower than at point  $x_1^r$ . Conventional gradient based procedures are therefore likely to fail at such points.

A procedure is needed at such points that will separate  $\omega_1$  from  $\omega_2$  while simultaneously separating  $\omega_1$  and  $\omega_3$ . Conventional gradient based procedures cannot provide this simultaneous separation.

Direct search optimization procedures avoid this difficulty since they do not involve derivatives. These procedures unfortunately are not



highly reliable [8] and the best of them is apparently not capable of treating the difficult six variable formulation of this shell design problem [3,9]. On the other hand a combination direct search and gradient based procedure, the Direct Search-Feasible Direction (DSFD) algorithm [10] which proved reliable on a series of 10 test problems of Ref. [8] also proved to be capable of treating this six variable shell problem [11]. The DSFD thus appears to be adaptable to the frequency separation problem. This procedure couples an efficient direct search scheme with a gradient based direction finding procedure used at points of direct search failure. A refined version of this procedure is used here [6].

As the direct search avoids the use of derivatives the objective function given by equation (5) can be used without difficulty at all points where direct search is utilized. At points of direct search failure however the search is restarted on the basis of local derivative information. Fortunately the direction finding algorithm used can easily be modified to handle the need to separate  $\omega_1$  and  $\omega_3$  as well as  $\omega_1$  and  $\omega_2$ .

The direction finding procedure of Zontendijk [12] is utilized to restart the basic direct search. A direction  $s_i$  in which an improved point may be found is normally determined from the solution of the linear programming problem:

Given the set  $x_i$ , find the set  $s_i$  that results in a

$$\max \sigma \quad (8)$$

for which

$$\sigma > 0 \quad (9)$$

$$(s_i)^T \nabla f(x_i) + \sigma < 0 \quad (10)$$



$$-(s_i)^T \nabla g_i(x_i) + W_j \sigma \leq 0 \quad j \in J_a \quad (11)$$

$$-1 \leq s_i \leq 1 \quad (12)$$

$$-1 \leq s_i \leq 0 \quad i \in -I_a^- \quad (13)$$

$$0 \leq s_i \leq 1 \quad i \in +I_a^+ \quad (14)$$

Here  $(s_i)^T$  indicates the transpose of vector  $s_i$ ,  $W_j$  is a weighting parameter, the set  $J_a$  contains the active constraints where

$$g_j(x_i) < e_1 \quad (15)$$

$e_1$  is small arbitrary positive constraint defining "activity", and  $I_a^-$ ,  $I_a^+$  constitute the active upper and lower regional constraints, respectively, where if

$$\left. \begin{array}{ll} x_i^u - x_i < e & i \in I_a^+ \\ x_i - x_i^l < e & i \in I_a^- \end{array} \right\} \quad (16)$$

Zoutendijk [12] shows that if the solution  $s_i$  is a null vector, then the point is a local optimum and if not then the direction of  $s_i$  is the best feasible direction.

Equations (8-14) constitute a linear programming problem with the variables  $s_i$  and  $\sigma$ . Equation (8) is the objective function and the remaining equations of the constraints. The solution  $s_i$  can be obtained reliably and efficiently using any suitable linear programming method.

At points where

$$\omega_3 - \omega_2 \leq e_3 \quad (17)$$

and  $e_3$  is an arbitrary small constant indicating frequency similarity equation (10) is replaced by the set

$$\left. \begin{aligned} (s_1)^T \nabla f_1(x_1) + \sigma &\leq 0 \\ (s_1)^T \nabla f_2(x_1) + \sigma &< 0 \end{aligned} \right\} \quad (18)$$

where  $f_1$  is given by eq (5) and

$$f_2 = \omega_1 - \omega_3 \quad (19)$$

It may be seen that the solution to the modified direction finding problem will yield a direction that will separate  $\omega_1$  and  $\omega_3$  as well as separating  $\omega_1$  and  $\omega_2$ .

In the event  $\omega_2 \approx \omega_3 \approx \omega_4$  the same modification procedure may be used. Thus if

$$\omega_4 - \omega_2 < e_3 \quad (20)$$

replace equation (10) with the set

$$\left. \begin{aligned} (s_1)^T \nabla f_1(x_1) + \sigma &< 0 \\ (s_1)^T \nabla f_2(x_1) + \sigma &< 0 \\ (s_1)^T \nabla f_3(x_1) + \sigma &< 0 \end{aligned} \right\} \quad (21)$$

where

$$f_3 = \omega_1 - \omega_4 \quad (22)$$

Similar reasoning may be adapted to treat problem type III of Ref. [1].

### III. The Shell Design Problem

An optimal shell design capability was developed along the lines of earlier procedures [3,4] to allow a study of the aforementioned optimal frequency separation method and to provide greater design flexibility than available in earlier programs.

Four problem types are treated:

#### Type 0 Problem

The objective function is

$$f(x_1) = W_D \quad (23)$$

where the design variables are as given in Fig. 1 and  $W_D$  is the weight/displacement ratio of the shell segment excluding the bulkheads [3].

The behavior constraints used control;

- $g_1$  = gross buckling
- $g_2$  = shell (inter-ring) buckling
- $g_3$  = shell yielding
- $g_4$  = stiffener yielding
- $g_5$  = stiffener flange buckling
- $g_6$  = maximum flange thickness
- $g_7$  = minimum flange width
- $g_8$  = minimum internal or maximum external radius
- $g_{11}$  = web buckling

constraints  $g_9$  and  $g_{10}$  are not used with the type 0 problem.

The equations used for  $g_3$ ,  $g_4$ ,  $g_6$ ,  $g_7$  and  $g_8$  are taken from [3].

Those used for  $g_5$  and  $g_{11}$  are essentially eqns (11) of Ref. [2]. Reference

[2] uses the same equations as [3] for the shell and stiffener yield constraints  $g_3$  and  $g_4$  except that [2] ignores the effect of eccentricity (out-of-roundness) on stiffener stress.

The basic behavior prediction equations for constraints  $g_1$  and  $g_2$  are adapted from Ref. [5] which uses a procedure described in [13]. Gross buckling control is achieved by stating that

$$g_1 = \frac{p_{cr}^* - FS_1 p}{p_{cr}^*} \geq 0 \quad (24)$$

where  $FS_1$  is the factor of safety for gross buckling  $p$  is the hydrostatic pressure and

$$p_{cr}^* = \min p_{cr}(n,m) \quad (25)$$

where  $n$  is the number of circumferential waves and  $m$  the number of axial half-waves. The equation for  $p_{cr}$  and the method for finding the minimum buckling load  $p_{cr}^*$  are given in Ref. [5]. This equation is based on the Donnell shell theory and a smeared stiffener orthotropic analysis.

The equation for the shell buckling constraint is similar to that for gross buckling except that  $x_5$ , the distance between stiffeners, replaces  $L_s$  the overall shell length in eqn (2) of Ref. [5] and the stiffener terms are omitted.

#### Type 1 Problem

This problem type is identical to type 0 except a minimum natural frequency constraint is used where

$$g_9 = \frac{\omega_1 - \omega_{\min}}{\omega_1} > 0 \quad (26)$$



where  $\omega_{\min}$  is the specified minimum frequency and  $\omega_1$  the lowest natural frequency of the structure. The quantity  $\omega_1$  is found from

$$\omega_1 = \min \omega(n,m) \quad (27)$$

where  $\omega$  is given by eqns (31)-(33) of Ref. [13].

The  $n$  and  $m$  producing the minimum frequency are located in exactly the same fashion as for the minimum buckling load. The minimum natural frequencies of both the entire shell and the segment of the shell between stiffeners are determined and the lowest value is used in eqn (27).

The frequency equations are derived using the same smeared stiffener orthotropic shell approach used for the general buckling equation. The effects of stiffener placement (interior or exterior) and torsional rigidity are considered but imperfection sensitivity is ignored. Experimental results indicate that such equations produce reasonably accurate results for hydrostatically loaded stiffened shells of the type studied here where the stiffener spacing is small compared to the buckling wave length as is the case for the range of parameters studied in this report. Imperfections do not play a major role in such shells [14,15].

It should be noted that the use of an in vacuo model for the vibration of shells submerged in water may produce substantial error [7,16]. Its use here however, is justified in light of the objectives of this work. These objectives are to present and evaluate the effectiveness of new methodology for the design of shells with optimal frequency separation and to gain insight into the characteristics of such shells.



A comparison of the values of the inter-ring buckling pressures produced by equation (8) of Ref. [3] and the procedure used here shows values within 2% for the designs of this study. Since the validity of the equation used in [3] has been demonstrated by experiment [17] the model used here for inter-ring buckling appears to be suitable even though it assumes simple support of the ends of the inter-ring panel. The effect of this assumption on the natural frequency values for such short shells is unknown. The use of this model for natural frequency prediction in this preliminary study, however, seems justified in light of the rather substantial error produced by the in vacuo assumption and the objectives of the study. A consideration of inter-ring vibration allows an opportunity to determine if this mode appears to be significant for the types of problems studied here. Furthermore inclusion of this vibration mode avoids the inconsistency in Ref. [1] wherein both an inter-ring buckling constraint (natural frequency is zero where such a constraint is active) and a minimum natural frequency constraint are specified [7].

#### Type 2 Problem

Here the objective function is given by equation (5) and a maximum weight constraint

$$g_{10} = \frac{W_{\max} - W_D}{W_D} \geq 0 \quad (28)$$

is used in addition to  $g_9$ , where  $W_{\max}$  is the specified maximum allowable weight/displacement ratio.

This is the optimal frequency separation problem (type II) of Ref. [1].

### Type 3 Problem

The objective in this problem type is to maximize the lowest natural frequency. Thus the objective function here is

$$f(x_1) = -\omega_1 \quad (29)$$

This problem type uses the same constraints as type 2.

A difficulty similar to that arising in problem type 2 is encountered here. The optimal search will encounter points where  $\omega_1 \approx \omega_2 \approx \omega_3$ . This situation is treated in the same fashion as for the type 2 problem. Thus where

$$\text{let} \quad \omega_2 - \omega_1 < e_3 \quad (30)$$

$$\left. \begin{array}{l} f_1 = -\omega_1 \\ f_2 = -\omega_2 \end{array} \right\} \quad (31)$$

and equations (18) are used in place of (10). Where

$$\omega_3 - \omega_1 < e_3 \quad (32)$$

let

$$f_3 = -\omega_3 \quad (33)$$

and replace eqn (10) by eqns (21).

Two additional options are available to the user of this capability. One option allows the designer to use a five variable formulation. Here the value of the web thickness is calculated so as to just satisfy the web buckling constraint. Thus web thickness is not treated as an independent variable when this option is selected. Since the web buckling constraint was found to be active in all cases studied this option can apparently be used to reduce computational effort while still producing optimal solutions.

Another option specifies a four variable formulation. Here the number of stiffeners to be used is specified rather than treated as a variable and the web thickness is calculated as above. Since it was found in earlier studies [3,4] that there is usually little increase in weight associated with specifying an arbitrary stiffener spacing, the designer is thus for practical purposes free to choose a stiffener spacing, within relatively broad limits, with little performance penalty for problem types 0 and 1.

The four variable formulation may be used with the type 2 and 3 problems but the impact on performance resulting from arbitrarily selecting stiffener spacing has not been studied.

#### IV. Results

A computer program called SBSHL7 coupling the DSFD optimization procedure and problems described above was used to repeat some of the shell design studies of Refs. [1,3] in order to evaluate the performance of SBSHL7 on this problem, to investigate the problem of optimal frequency separation, and to allow a comparison of the results generated with those of Refs. [1,3]. All studies use the following design parameters unless otherwise indicated; shell midplane radius  $R = 5.029\text{m}$  (198 in.); shell length  $L_s = 15.09\text{m}$  (594 in.) shell eccentricity of zero; immersion depth 304.8m (1000 ft); specific weight of immersion fluid (sea water)  $\gamma_w = 1.0256 \text{ g/cm}^3$  (0.03741lb/in.<sup>3</sup>), specific weight of shell material  $\gamma_s = 7.733 \text{ g/cm}^3$  (0.282 lb/in.<sup>3</sup>) Young's modulus  $20.68 \times 10^6 \text{ N/cm}^2$  ( $30 \times 10^6 \text{ psi}$ ), Poisson's ratio  $\mu = 0.30$  and allowable yield stress of  $41,360 \text{ N/cm}^2$  (60,000 psi) for the shell and stiffener material. Factors of safety of 2 are used for all buckling constraints and a minimum natural frequency of zero is specified. All shells use interior stiffeners.

Table 1 demonstrates the reliability of the new optimization procedure on the type 0 problem. The starting points, in mm, used for this study were:

$$(x_1^0)_1 = (0,0,0,0,0,0)$$

$$(x_1^0)_2 = (12.7,12.7,12.7,127,254,127)$$

$$(x_1^0)_3 = (25.4,25.4,25.4,254,508,254)$$

$$(x_1^0)_4 = (38.1,38.1,38.1,381,762,381).$$



Table 1. Convergence by SBSHL7 to an Optimal Configuration  
Widely Separated Starting Points for the Type 0 Problem.

	Ref. [8]	Starting Point 1	Starting Point 2	Starting Point 3	Starting Point 4
$W_D$ , weight/ displacement ratio	0.1357	0.10306	0.10303	0.10313	0.10312
$x_1$ , skin thickness, mm (in)	26.901 (1.0591)	28.092 (1.1060)	28.128 (1.074)	28.082 (1.1056)	28.067 (1.1050)
$x_2$ , web thickness, mm (in)	24.226 (0.9538)	6.7158 (0.2644)	6.6751 (0.2628)	6.7539 (0.2659)	6.7005 (0.2638)
$x_3$ , flange thickness, mm (in)	9.9772 (0.3928)	33.609 (1.3232)	27.277 (1.0739)	12.893 (0.5076)	15.133 (0.5958)
$x_4$ , flange width, mm (in)	125.308 (4.933)	25.293 (0.9958)	30.322 (1.1938)	67.774 (2.6683)	55.535 (2.1864)
$x_5$ , stiffener spacing, mm (in)	580.29 (22.846)	405.79 (15.9763)	407.67 (16.050)	412.47 (16.239)	406.30 (15.996)
$x_6$ , web height, mm (in)	463.07 (17.168)	292.68 (11.5238)	296.93 (11.690)	299.14 (11.777)	298.37 (11.747)
$g_1$ , gross buckling	0.622	0.000	0.000	0.000	0.000
$g_2$ , panel buckling	0.003	0.440	0.438	0.423	0.437
$g_3$ , skin yield	0.004	0.000	0.000	0.000	0.000
$g_4$ , stiffener yield	0.339	0.088	0.088	0.089	0.089
$g_5$ , flange buckling	0.003	1.000	0.993	1.000	0.989
$g_6$ , web buckling	*	0.041	0.001	0.013	0.000

\*Web buckling controlled by setting  $x_6 = 18 x_3$ .



It may be seen that a series of runs from widely separated starting points converged to similar designs with weights that are within 0.15% of the lowest. This behavior is indicative of reliable optimization algorithm performance and the absence of local optima.

The designs generated by SBSHL7 are substantially lighter than those given in Ref. [3] due primarily to the elimination of an unrealistic stiffener buckling constraint used in [3] but not used here or in [1,5].

The shell buckling equations used here are considerably more complex than those of [3]. This added complexity in addition to the increase in problem dimensionality and difficulty resulting from the uncoupling of the stiffener variables greatly increases the computational effort required for solution. The program SBSHL7 typically requires about 100 times more CPU time than that of [3]. Thus, the added sophistication is obtained at substantially increased cost. Still, if a reasonable starting point is specified, the cost of solution is not excessive (normally less than 2 min CPU time on an IBM 370-168). The buckling equations used in [3] can unfortunately lead to invalid designs for certain ranges of parameter values [5] since the interior buckling minimum ( $m > 1$ ) is not considered. This is also true of the procedure of Ref. [1] where  $m > 6$  is ignored. Thus, this added complexity is justified. Fortunately the computational cost can apparently be drastically reduced by use of a five variable formulation as will be shown below.

Table 2 demonstrates the superiority of DSFD over the SUMT procedure used in [1] which failed to locate the minimum, producing designs substantially heavier than those presented here. The shell parameters utilized in [1]

Table 2. Comparison of Designs Developed by SBSHL7  
With Those of Refs. [1,2] for the Type 1 Problem.

	Ref. [1]	Ref. [2]	SBSHL7
$W_D$ , weight/ displacement ratio	0.13317	0.11274	0.10746
$x_1$ , skin thickness, mm (in)	30.754 (1.2108)	30.622 (1.2056)	29.522 (1.1623)
$x_2$ , web thickness, mm (in)	9.563 (0.3765)	6.0274 (0.2373)	5.1054 (0.2010)
$x_3$ , flange thickness, mm (in)	11.951 (0.4705)	7.8003 (0.3071)	4.6533 (0.1832)
$x_4$ , flange width, mm (in)	448.66 (17.664)	263.22 (10.363)	150.80 (5.937)
$x_5$ , stiffener spacing, mm (in)	853.49 (33.602)	766.32 (30.170)	424.51 (16.713)
$x_6$ , web height, mm (in)	497.56 (19.589)	279.91 (11.020)	239.04 (9.411)
$g_1$ , gross buckling	Not given	Not given	0.234
$g_2$ , panel buckling	Not given	Not given	0.742
$g_3$ , skin yield	Not given	Not given	0.000
$g_4$ , stiffener yield	Not given	Not given	0.081
$g_5$ , flange buckling	Not given	Not given	0.039
$g_6$ , web buckling	Not given	Not given	0.000
$g_7$ , min. nat. freq. $H_z$	Not given	Not given	0.000
$\omega_1$ , $H_z$	28.12	12.03	12.00
$\omega_2$ , $H_z$	49.39	22.30	20.50

are used for this study. They are identical to those given above except that  $\mu = 0.33$ ,  $\gamma_g = 8.225 \text{ g/cm}^3$  ( $0.30 \text{ lb/in.}^3$ ),  $\omega_{\min} = 12 \text{ Hz}$ , and factors of safety equal to unity are used for the buckling constraints.

The design of Ref. [1] is obviously not optimal since an optimal design should converge to the minimum frequency constraint. Neither is the design a local optimum. This design was used as a starting point for a synthesis run using SBSHL7. The search immediately located a better nearby design and moved to a design essentially identical to that presented in Table 2 for SBSHL7. Since the general and shell buckling constraints are not active at this point, the constraint equations used here are identical to Ref. [1]. Thus, the design of Ref. [1] does not appear to be a local optimum.

The design presented in Table 2 generated by SBSHL7 used the starting point  $(x_1^0)_1$ .

The design generated in Ref. [2] cannot be considered as representative of the performance of the SUMT procedure of [1] since it used a near optimal starting point supplied by Pappas [7]. The performance of SUMT in [1,2] coupled with the evidence developed in an earlier comparison study [8], strongly suggests that SUMT simply cannot cope with this six variable shell design problem.

Problem type 2 and 3 were run using both five and six variable formulations. Table 3 give the results of typical problem type 2 and 3 runs. A maximum weight displacement ratio,  $(W_D)$  of 0.150 was used for these runs. The quantities  $n_f$  and  $n_g$  are the number of objective and constraint function evaluations required for convergence. Several observations can be made from the data contained in this table.



Table 3. Optimal Designs for Problem Type 2 and 3

	Initial Point	Type 2, 5 Var. Optimum	Type 2, 6 Var. Optimum	Type 3, 6 Var. Optimum	Type II Ref. [1]
$W_D$	0.155	0.150	0.150	0.150	0.135
$x_1$	25.40 (1.000)	38.46 (1.514)	38.56 (1.518)	35.00 (1.378)	31.03 (1.222)
$x_2$	25.40 (1.000)	13.97 (0.550)	13.69 (0.539)	11.81 (0.465)	10.03 (0.395)
$x_3$	25.40 (1.000)	15.29 (0.602)	33.81 (1.331)	23.04 (0.907)	11.82 (0.465)
$x_4$	254.0 (10.000)	246.4 (9.702)	122.6 (4.826)	541.0 (21.30)	445.8 (17.55)
$x_5$	508.0 (20.000)	1396 (54.96)	1398 (55.04)	1372 (54.00)	859.9 (33.85)
$x_6$	254.0 (10.000)	820.9 (32.32)	804.9 (31.69)	731.0 (28.78)	526.3 (20.72)
$g_1$	0.539	0.789	0.790	0.749	0.677
$g_2$	0.472	0.304	0.307	0.126	0.342
$g_3$	0.001	0.114	0.116	0.001	-0.002
$g_4$	0.389	0.467	0.466	0.521	0.325
$g_5$	0.947	0.868	0.994	0.714	0.619
$g_6$	0.000	0.602	0.113	0.341	0.018
$g_7$	10.000	9.702	4.826	21.998	17.551
$g_8$	0.598	0.544	0.543	0.553	0.574
$g_9$	0.534	0.717	0.717	0.743	0.673
$g_{10}$	-0.035	0.001	0.001	0.000	0.120
$g_{11}$	0.967	0.000	0.000	0.005	-0.006
$\omega_1(n_1 m_1)$ Hz	25.76(2,1)	42.40(2,1)	42.41(2,1)	46.71(2,1)	28.37(36.65)*
$\omega_2(n_2 m_2)$ Hz	33.04(3,1)	72.84(1,1)	72.90(3,1)	46.87(14,1)*	51.96(58.52)**
$\omega_3(n_3 m_3)$ Hz	44.29(3,2)	72.84(3,1)	72.90(1,1)	47.50(15,1)*	51.96
$\omega_4(n_4 m_4)$ Hz	47.82(1,1)	72.88(14,1)*	73.19(14,1)*	49.83(13,1)*	-
$(\omega_2 - \omega_1)$ Hz	7.28	30.44	30.49	0.16	23.59(21.86)**
$n_f$	1	2963	16.779	22,572	-
$n_g$	11	2118	34,371	50,193	-

\*Mode associated with shell (inter-ring) vibration.

\*\*Frequency using Donnell shell theory employed in this report.

The computational effort required to achieve convergence using the five variable formulation is substantially less than that using the six variable form. Since all studies of all problem types indicates that the web buckling constraint is always active at the optimum there seems to be little reason to use the six variable formulation for design purposes. The principal advantage of this form is that it provides a costly check of the assumption, used in the five variable form, that web buckling is active at the optimum.

An examination of the optimal design for the type 2 problem reveals several interesting characteristics. Not only is  $\omega_3$  essentially equal to  $\omega_2$  as found in Refs. [1,2] but  $\omega_4$  is also essentially equal to  $\omega_2$ . Furthermore,  $\omega_4$  is associated with shell panel (inter-ring) vibration. The design is characterized by relatively large frame spacing and large, deep framing members. Thus, the optimal design uses the largest frame members and spacing possible without inducing a shell panel vibration mode lower than the second frequency associated with the gross vibration, and without violating the maximum weight constraint. It is apparent therefore that the shell panel vibration should be considered in optimal frequency separation problems.

Since the designs are controlled by the maximum weight constraint it appears that optimal separation is achieved by paying a penalty in weight.

The characteristics of the design with the maximum lowest frequency is rather similar to that with the largest frequency separation. The former has a somewhat higher lowest frequency but no significant separation of this frequency from the second or third lowest frequencies. Here again, shell panel vibration controls the design.



Comparing the designs in table 3 it appears that rather similar designs may behave quite differently with respect to frequency separation. This raises a serious question with regard to the validity of frequency separation data. Orthotropic shell theory is only an approximation to actual behavior. This and the earlier study of Refs. [1,2] furthermore use in vacuo frequencies to study the characteristics of submerged shells. Considering the inaccuracy of these assumptions and the sensitivity of the frequency separation results to relatively small design changes the data for the type 2 problem appear to have little meaning except to indicate possible general characteristics of such designs.

In addition to the search starting point cited in Table 3 were made using the six variable formulation for problem types 2 and 3 from the three additional starting points used in the type 0 study. These also converged to designs similar to those of table 3 thus the starting point sensitivity noted in Ref. [1] on their problem type 2 was not apparent here. These results indicate that the optimization procedure used here is apparently capable of locating an optimum design with reasonable reliability for all problem types studies.

It should be noted that the frequency separation obtained for the type 2 problem ( $\omega_2 - \omega_1 = 30.48$  Hz) is substantially greater than obtained in Ref. [1] ( $\omega_2 - \omega_1 = 23.59$  Hz) and that Ref. [1] failed to converge to the maximum weight constraint. It appears therefore that the SUMT procedure used in [1] also failed to locate the optimum design in the type 2 problem of Ref. [1]. The difference in frequency separation results do not appear to be due to the difference in shell theory used in [1] (Flugge-Lure-Byrne) with that used here (Donnel type). The point cited as optimal in [1] produced

a separation of 21.86 Hz using the Donnel theory utilized here, a value somewhat lower than that of [1], but still in good agreement. This lower value further suggests that the frequency separation at the optimal point claimed here, would in fact be somewhat larger using the frequency prediction procedure of [1].

#### IV. Conclusion

Design for optimal frequency separation may require treatment of the case where several frequencies must be separated simultaneously. The methodology presented here appears effective in treating such problems. These studies indicate that the simultaneous separation procedure and DSFD optimization algorithm utilized here are capable of reliably treating the optimal frequency separation problem.

The design studies show that shell panel vibration modes may be active at the optimum. Thus, a consideration these modes should be included in the optimal frequency separation problem. The similarity of designs with optimal and negligible separation of the two lowest natural frequencies as predicted by the orthotropic in vacuo model used here and elsewhere indicates that more accurate behavior prediction models are needed to produce meaningful design results.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NJIT Report No. NV-11	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Optimal Frequency Separation In Cylindrical Shells	5. TYPE OF REPORT & PERIOD COVERED Interim rept.	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Michael Pappas	8. CONTRACT OR GRANT NUMBER(s) ONR-N-00014-75-C-0987	9. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 12 29p.
10. PERFORMING ORGANIZATION NAME AND ADDRESS New Jersey Institute of Technology 323 High Street Newark, New Jersey 07102	11. CONTROLLING OFFICE NAME AND ADDRESS Dr. Nicholas Perrone, (Code 474) Office of Naval Res., 800 No. Quincy St., Arlington, Virginia 22203	12. REPORT DATE March 1977
13. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	14. SECURITY CLASS. (of this report) Unclassified	15. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Distribution of this Document is unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Vibration, Shells; Optimization, Shells; Automated Design; Optimal Design.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  A new and powerful algorithm is applied to the problem of optimal frequency separation of stiffened cylindrical shells. New methodology is introduced which allows the simultaneous separation of several modes of vibration. Results indicate that the procedure can effectively deal with this difficult optimization problem.		

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